

Answer all questions:

Q1 – Given the following system [5]

$$y(t) = \int_{\tau=a}^t \left(\frac{\tau}{t} \right) \cdot x(\tau) d\tau$$

Check for time invariant & linearity

Q2 – Consider the continuous-Time system $(D^2 + 2D - 2)[y(t)] = 5x(t)$ [5]

- Evaluate the frequency response of the system.
- Find the output of the system when

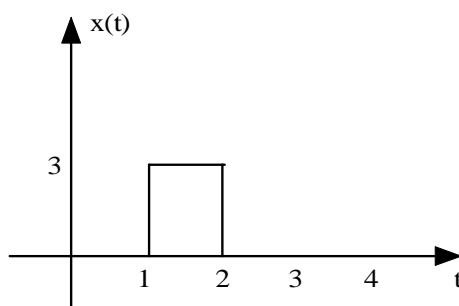
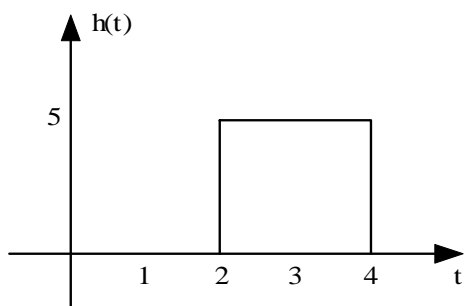
$$x(t) = e^{-3t} \quad t \geq 0, \quad y(0) = 1, y^{(1)}(0) = 0.$$

Q3 – Given the following transfer function [5]

$$H(z) = \frac{6(s + 34)}{s(s^2 + 10s + 34)}$$

Evaluate the inverse of the given Laplace transform

Q4 - When the input to a continuous-Time system $\delta(t)$ the output is the shown $h(t)$ find the output of the same system to the given input signal $x(t)$. [5]



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Solution of Q1

$$y(t) = \int_{\tau=a}^t \left(\frac{\tau}{t} \right) \cdot X(\tau) d\tau$$

$$y(t)_1 = \int_{\tau=a}^t \left(\frac{\tau}{t} \right) \cdot X(\tau - t_0) d\tau$$

$$U := \tau - t_0$$

$$d\tau = du$$

$$y(t)_1 = \int_{u=a-t_0}^{u=t-t_0} \left(\frac{u+t_0}{t} \right) \cdot X(u) du$$

which is not equal to $y(t-t_0)$ -----> time varying

$$Q2) \quad H(j\omega) = \frac{5}{(j\omega)^2 + 2(j\omega) - 2}$$

$$y^h(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t} \quad r_1 = -2.7321, \quad r_2 = 0.7321$$

$$y^p(t) = 5e^{-3t}$$

$$y(t) = y^h(t) + y^p(t) = -5.1753e^{r_1 t} + 1.753e^{r_2 t} + 5e^{-3t} \quad t \geq 0$$

Q3)

$$H(s) = \frac{6(s+34)}{s(s^2+10s+34)} = \frac{6}{s} - \frac{3-4i}{s+5-3i} - \frac{3+4i}{s+5+3i}$$

$$k = -3 - 4i = 5 \angle -126.869$$

$$f(t) = [6 + 2 | 5 | e^{-5t} \cos(3t + 126.9)] u(t)$$

$$H(s) = \frac{6(s+34)}{s(s^2+10s+34)} = \frac{6}{s} - \frac{6(s+5)}{(s+5)^2+9} - \frac{8(3)}{(s+5)^2+9}$$

$$f(t) = [6 - 6e^{-5t} \cos(3t) - 8e^{-5t} \sin(3t)] u(t)$$

Q4)

$$y(t) = x(t) * h(t)$$

